THE VARIATION OF ICE STRENGTH WITHIN AND BETWEEN MULTIYEAR PRESSURE RIDGES IN THE BEAUFORT SEA

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ABSTRACT

A recent series of tests on the uniaxial compressive strength of ice samples taken from multiyear pressure ridges allows the testing of several hypotheses concerning the variation in strength within and between ridges. The data set consists of 218 strength tests performed at two temperatures (-5° and -20°C) and two strain rates (10^{-3} and 10^{-5} s^{-1}). There was no significant difference between the strength of the ice from the ridge sails and the ice from the ridge keels when tested under identical conditions. As the total porosity of the ice from the sails is higher by 40% than the ice from the keels, the lack of a significant difference is believed to result from the large variations in the structure of the ice which occur randomly throughout the cores.

A three-level analysis of variance model was used to study the variations in strength between 10 different ridges, between cores located side by side in a given ridge, and between samples from the same core. In all cases the main factor contributing to the observed variance was the differences within cores. This is not surprising considering the rather extreme local variability in the structure of ice in such ridges. There was no reason at the 5% level of significance to doubt the hypothesis that the different cores at the same site and the different ridges have equal strength means.

INTRODUCTION

When a pressure ridge forms, it is a poorly consolidated mass of sea ice blocks and slabs intermixed with snow and slush. As it ages, its overall salinity decreases, and bonding between the blocks increases, resulting in an increase in the overall strength of the ridge. When surface melting starts in the spring, the relatively low salinity and low density meltwater percolates downward into the core of the ridge, where it displaces seawater that is either near or at its freezing point of approximately -1.8°C. As a result, much of the percolating meltwater freezes, welding the ice blocks together and gradually filling the voids with new, low-salinity ice. If the ridge survives the summer and the meltwater-filled voids refreeze, a multiyear pressure ridge is produced. Such ridges show the combined characteristics of great thickness (values in excess of 30 m have been observed), low salinity (usually less than 3.5 o/oo) and low porosity (1,2).

As multiyear ridges are quite common, even in the nearshore regions of the Beaufort and Chukchi seas, it is hardly surprising that their properties are important in estimating the peak forces that the pack ice might exert on an offshore structure emplaced in deeper water off the north coasts of Alaska and Canada. Considering the obvious importance of these ice features, it is, at first glance, surprising that so little effort has been devoted to studying the properties of either multiyear ice or multiyear ridges. The reason for this neglect was that the study of multiyear ice required heavy logistical support to reach suitable sampling locations. This was, of course, expensive and outside the range of most research budgets prior to the discovery of major oil and gas resources in the Arctic. In addition there was a natural tendency to avoid the study of presumably more complex multiyear ice features until a reasonably adequate understanding of property variations in simpler first-year ice was achieved.

The two preceding papers attempt to partially fill the data gap on the compressive strength and structure of ice from multiyear pressure ridges (3,4). In this third paper the data set is examined further in order to understand the sources of the large variations in ice strength. Specifically, the purpose was to determine if there were any significant differences in ice strength between samples collected from the ridge sails and keels and if there were any consistent trends of ice strength versus depth, given that the ice was all tested under identical conditions. In addition, an assessment was made of the variance in ice strength between ridges, between cores located side by side on a given ridge, and between

samples from the same core. Histograms were also prepared to examine the frequency distribution of ice strengths at each of the four test conditions.

YIELD STRENGTH DIFFERENCES BETWEEN RIDGE SAILS AND KEELS

It would appear reasonable that ice from the above-sea-level portions of multiyear ridges (the ridge sails) might show higher yield strengths than ice from the below-sea-level portions (the ridge keels); studies have shown that ridge sails have consistently lower salinities than ridge keels (1,2). Therefore, at a given temperature, sail ice would have a lower brine volume and a higher strength than keel ice.

To test this hypothesis, when each core was obtained from a ridge the elevation of the top of the core relative to the upper surface of the surrounding level ice was determined. This allowed the ice in each core to be classified as above level ice and below level ice, a classification that approximately corresponds to above sea level and below sea level (the level ice elevations in the study area would not be expected to vary by more than 0.2 m from sea level). Using this basic division of samples there are four sets of data that can be tested for differences, as the ridge ice samples were tested at two strain rates (10^{-3} and 10^{-5}/s) and two temperatures (-5° and -20°C). Table 1 gives a summary of the properties of these data sets with each set subdivided into above-level-ice and below-level-ice portions. The hypothesis that is tested is whether or not there is any reason, based on the available data, to doubt that the above- and below-level-ice samples have the same yield strength population means (i.e., H_o: $\mu_a = \mu_b$, where μ is the population mean and the subscripts a and b indicate above and below level ice).

In all four cases, even if we were to accept a 20% chance of being incorrect, it was found that based on a two-tailed t-test there is no reason to doubt that both the above- and below-level-ice samples have the same population means. It is interesting to speculate about the reasons for this result. As was expected the average salinity of the ice from the ridge sails proved to be lower by 0.8 °/oo than the salinity of the ice from the ridge keels. This by itself would cause the keel ice to be weaker. However, this proved to be offset by a higher gas volume in the ridge sails. In fact, the total porosity (gas + brine) of the sail ice is significantly higher by roughly 40% than the porosity of the keel ice. This, of course, (see Figure 4 in (3)) should result in the sail ice being weaker. It is believed that the fact that such a trend is not discernible is caused by the large variations in ice strength that are produced by changes in the internal structure of the ice. As these structural changes occur essentially at random throughout an ice core and are not related to the location of a sample relative to sea level, they tend to obscure any differences that exist between the strength of the ice in the upper and lower portions of multiyear ridges. This absence of a discernable difference between the above- and below-level-ice samples is important, as we can now combine both the above- and below-level-ice samples into one population in the Analysis of Variance (AOV) that follows.

The variation in strength with vertical position in a core has also been examined in another way. For each of the 74 cores from which 2 or more samples were obtained, a plot was made of strength versus the depth of the sample measured below the upper ice surface. Figure 1 is an example of these plots. For each core the slope of the linear regression line of strength versus depth was then determined. Figure 2 shows a frequency histogram of the resulting slope values. As can be seen, the histogram is symmetrical with a mean of essentially zero. There clearly is no reason to believe that there is a systematic variation in strength with depth in the sampled multiyear pressure ridges. This, of course, does not mean that the upper and lower portions of in situ ridges necessarily have

Table 1. Statistical characteristics of the uniaxial compression strength of the samples from above and below level ice. Symbols are as follows: $\bar{\sigma}_c$ = average; s = standard deviation; n = number of tests; t = value of the t-test for differences between means. Strength values are in lbf/in ^{2} and (MPa).

Test conditions	Above level ice $\bar{\sigma}_c$	Above level ice s	Above level ice n	Below level ice $\bar{\sigma}_c$	Below level ice s	Below level ice n	Difference between means	t	t for 0.05 significance level	t for 0.20 significance level
-5°C (23°F)										
$\dot{\varepsilon} = 10^{-5}$/s	338 (2.33)	140 (0.97)	21	343 (2.36)	170 (1.17)	48	5 (0.03)	0.11	2.00	1.29
$\dot{\varepsilon} = 10^{-3}$/s	837 (5.77)	236 (1.63)	25	902 (6.22)	240 (1.65)	44	65 (0.45)	1.10	2.00	1.29
-20°C (-4°F)										
$\dot{\varepsilon} = 10^{-5}$/s	428 (2.95)	106 (0.73)	15	379 (2.61)	121 (0.83)	24	49 (0.34)	1.26	2.03	1.30
$\dot{\varepsilon} = 10^{-3}$/s	1425 (9.83)	227 (1.57)	15	1377 (9.49)	187 (1.29)	26	49 (0.34)	0.72	2.03	1.30

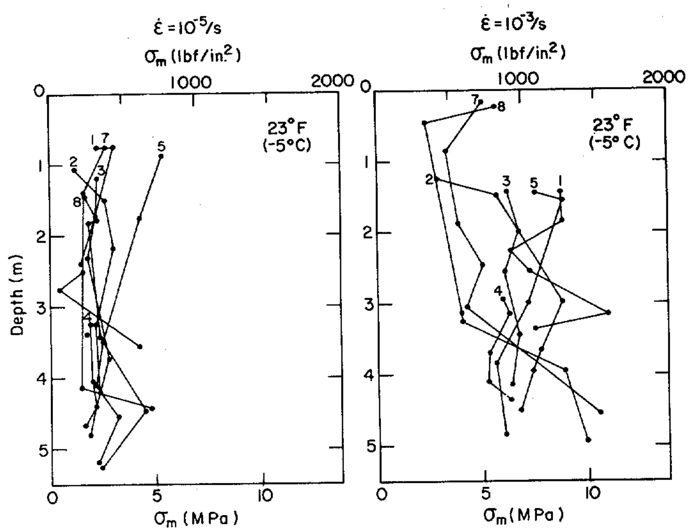

Figure 1. Uniaxial compressive strength vs depth for a number of multiyear pressure ridge cores. The number at the top of each profile is the ridge number from which the samples were obtained.

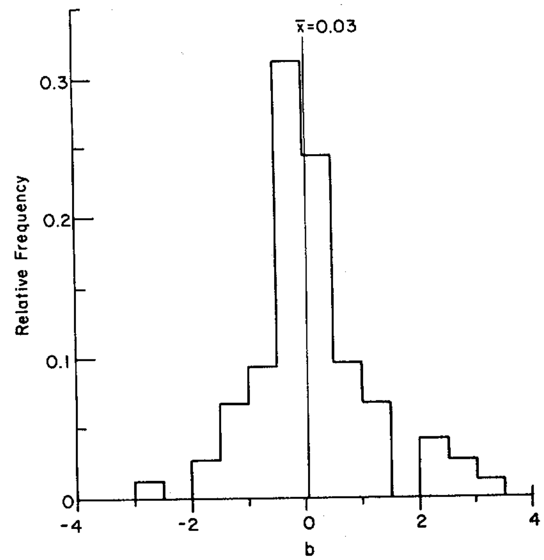

Figure 2. Frequency histogram of regression line slopes of strength vs depth.

the same strength; during the ice growth season the near-surface ice is commonly stronger because of its lower temperature.

SOURCES OF THE VARIATION IN STRENGTH

It was initially planned to collect test samples from exactly the same levels in collocated cores (i.e. located as close together as practical) from each ridge. This did not prove possible because of problems with gouges and breaks in the cores. Instead, because of the erratic location of the gouges in each core, the vertical locations of the samples in each core were approximately random. This, coupled with the fact that there was no systematic difference between the strength values of the above- and below-level-ice samples, makes it possible to study the observed strength variation by using a three-level Analysis of Variance (AOV) model (5). In this model the total sample variance is partitioned into the variance components contributed by differences

a) between ridges,
b) between cores collocated on a given ridge, and
c) between samples from the same core.

The linear AOV model assumed is

$$k_{ijk} = \mu + v_i + y_{ij} + z_{ijk}$$

with $i = 1 \ldots r$, $j = 1 \ldots t$, and $k = 1 \ldots n$. Here μ is the grand mean, v_i corresponds to the ridge effect, y_{ij} to the effect of collocated cores within the same ridge, and z_{ijk} to the effect of samples within the same core. The parameters v_i, y_{ij} and z_{ijk} are assumed to be normally distributed with zero means and variances ψ^2, ω^2 and σ^2, respectively. Table 2 gives the computational relations for this model, and Table 3 gives the results for the four test conditions. In Table 3 we have used data only when a complete set of three samples (one above level ice and two below level ice) were available for a given core. Because of breakage and gouging this reduces the number of degrees of freedom between ridges to between

Table 2. Analysis for a three-level nested AOV model.

Source of Variance	Sums of Squares	Degrees of Freedom	Mean Squares	E[Mean Squares]
Between Ridges	$nt\sum_{i}^{r}(\bar{x}_{i..}-\bar{x}_{...})^2$	r-1	s_r^2	$\sigma^2 + n\omega^2 + nt\psi^2$
Between collocated cores within a ridge	$n\sum_{ij}^{rt}\sum(\bar{x}_{ij.}-\bar{x}_{j..})^2$	r(t-1)	s_t^2	$\sigma^2 + n\omega^2$
Between samples from the same core	$\sum_{ijk}^{rtn}\sum\sum(x_{ijk}-\bar{x}_{ij.})^2$	rt(n-1)	s_n^2	σ^2
TOTAL	$\sum_{ijk}^{rtn}\sum\sum(x_{ijk}-\bar{x}_{...})^2$	rtn-1		

Table 3. Results of a three-level nested AOV analysis of the variation in compressive strengths (values given in lbf/in^2 and (MPa)). Cores are used only if there are no "missing" samples. There are n samples from each core, t co-located cores on each ridge, and r ridges.

	Source of Variation	Sum of Squares	d.f.	Mean Squares	Expected Mean Squares	r,t,n	$\hat{\sigma}$	$\hat{\omega}$	$\hat{\Psi}$	$H_o: \omega^2=0$ F	$H_o: \omega^2=0$ $F_{0.95}$	$H_o: \Psi^2=0$ F	$H_o: \Psi^2=0$ $F_{0.95}$
-5°C (+23°F)													
10^{-5}/s	Between ridges	247,178 (11.75)	4	61,795 (2.94)	$\sigma^2+3\omega^2+6\Psi^2$	5,2,3	180 (1.24)	90 (0.62)	29 (0.20)	1.74	2.71	1.10	5.19
	Between cores at a site	283,245 (13.47)	5	56,651 (2.69)	$\sigma^2+3\omega^2$								
	Between samples within cores	650,369 (30.92)	20	32,518 (1.55)	σ^2								
10^{-3}/s	Between ridges	118,693 (5.64)	2	59,347 (2.82)	$\sigma^2+3\omega^2+6\Psi$	3,2,3	412 (2.84)	210 (1.45)	61 (0.42)	0.22	3.49	1.61	9.55
	Between cores at a site	110,852 (5.27)	3	36,951 (1.76)	$\sigma^2+3\omega^2$								
	Between samples within cores	2,034,966 (96.7)	12	169,581 (8.06)	σ^2								
-20°C (-4°F)													
10^{-5}/s	Between ridges	48,027 (2.28)	3	16,009 (0.76)	$\sigma^2+3\omega^2+6\Psi$	4,2,3	127 (0.88)	69 (0.48)	119 (0.82)	0.11	3.01	8.80*	6.59
	Between cores at a site	7,276 (0.35)	4	1,819 (0.09)	$\sigma^2+3\omega^2$								
	Between samples within cores	259,603 (12.34)	16	16,225 (0.77)	σ								
10^{-3}/s	Between ridges	279,144 (13.27)	2	139,572 (6.63)	$\sigma^2+3\omega^2+6\Psi$	3,2,2	209 (1.44)	50 (0.34)	121 (0.83)	1.17	3.49	2.72	9.55
	Between cores at a site	154,017 (7.32)	3	51,339 (2.44)	$\sigma^2+3\omega^2$								
	Between samples within cores	525,533 (24.98)	12	43,794 (2.08)	σ^2								

*Significant at the 5% level. However, not significant at the 1% level, as $F_{0.99}$ = 16.69.

2 and 4. The results indicate that in all cases there is no reason to doubt the hypothesis that ω^2 equals zero (i.e., there is no significant variation between cores at the same site). Also, in three of the four test situations there is no reason to doubt the hypothesis that ψ^2 equals zero (i.e., that there is no significant difference between ridges).

Several cores in each data set were, for a variety of reasons, missing one sample. To avoid discarding the two samples in each core with a missing sample (as we did in the previous analysis), we also completed an approximate analysis in which we replaced each of the missing values with the mean of the other observations from the same core. The inserted values therefore made no contribution to the residual sum of squares. This analysis indicates that in all cases there is no reason to doubt the hypotheses that there is no significant variation between cores at the same site and that there is no significant variation between ridges. Detailed AOV tables for these tests can be found in Cox et al. (6). Table 4 summarizes the differences in the results of the two analyses. The main factor contributing to the observed variance is associated with differences within cores. This is not surprising, considering the extreme local variability in the structure of the ice in multiyear pressure ridges (the variance between cores at a site and between ridges was always much less than that within cores). In more than 50% of the cases, however, the variance associated with differences between ridges was larger than that observed between cores in the same ridge. Again these results are reasonable. In some ridges where the block sizes are either large or very small, it might be expected to obtain a low variance from collocated cores. In other ridges where the blocky structure is intermediate in size, a higher variance would presumably occur.

These results do not mean that we believe that all multiyear pressure ridges have identical strengths. As a first-year ridge is gradually transformed into a multiyear ridge, the voids in the ridge are slowly sealed with ice, presumably increasing the bulk strength of the ridge. In fact, one of the ridges sampled (ridge 6) contained many large voids, which caused the core recovery to be so poor that we moved to another ridge. We believe that this ridge had been through only one melt season, and as a result many of the voids had not rehealed. We have also sampled a ridge (not included in the present data set) that contained many large gas bubbles. The ridges included in the present data set had well-rounded surface profiles and are believed to be several years old. Also, in several of the ridges, we were able to examine the surfaces of fractures traversing the ridge

Table 4. Summary of differences in the data sets and AOV results between the cases, (a) when no values are missing and (b) when average values are substituted for missing values. Strength values are in lbf/in ^{2} and (MPa).

Test Temperature	Strain Rate (s^{-1})	No. of Missing Values	No. of Ridges	Estimated Standard Deviation: Within Cores, $\hat{\sigma}^2$	Between Cores at a Site, $\hat{\omega}^2$	Between Ridges, $\hat{\psi}^2$	$H_o: \omega^2=0$ F	$H_o: \omega^2=0$ $F_{0.95}$	$H_o: \psi^2=0$ F	$H_o: \psi^2=0$ $F_{0.95}$
-5°C	10^{-5}	0	5	180 (1.24)	90 (0.62)	29 (0.20)	1.74	2.71	1.10	5.19
		2	7	169 (1.17)	71 (0.49)	43 (0.30)	1.52	2.36	1.25	3.87
	10^{-3}	0	3	412 (2.84)	210 (1.45)	61 (0.42)	0.22	3.49	1.61	9.55
		4	7	288 (1.99)	114 (0.79)	131 (0.90)	0.53	2.36	3.35	3.87
-20°C	10^{-5}	0	4	127 (0.88)	69 (0.48)	119 (0.82)	0.11	3.01	8.80*	6.59
		4	6	106 (0.73)	37 (0.26)	42 (0.30)	0.65	2.51	2.44	4.39
	10^{-3}	0	3	209 (1.44)	50 (0.34)	121 (0.83)	1.17	3.49	2.72	9.55
		5	6	171 (1.18)	45 (0.31)	129 (0.89)	1.21	2.51	2.41	4.39

* Significant at the 5% level, however, this is not significant at the 1% level where $F_{.99}(3,4) = 16.69$.

in order to ascertain that the ridge was composed of massive ice that was nearly void-free. Therefore, we believe that our data set is reasonably representative of old, solid, well-healed pressure ridges and that even in these ridges the homogenization processes associated with aging are not sufficient to erase the large differences in mechanical properties caused by local structural differences within the ice.

SHAPE OF THE STRENGTH HISTOGRAMS

Histograms were also prepared for examining the frequency distribution of ice strength at each of the four test conditions. Figure 3 shows histograms based on the four data sets, and Table 5 presents the first four moments (μ_1, μ_2, μ_3 and μ_4), the skewness (α_3) and the kurtosis (α_4) for each data set. For symmetrical distributions such as the normal, $\alpha_3 = 0$. The kurtosis is a measure of the peakedness of the distribution; if $\alpha_4 = 3$, the peakedness corresponds to that of a normal distribution, with higher values indicating a distribution that is more peaked than normal and lower values indicating a distribution that is broader than normal. At the higher strain rates (10^{-3}/s) both sets of data show a positive skew, but

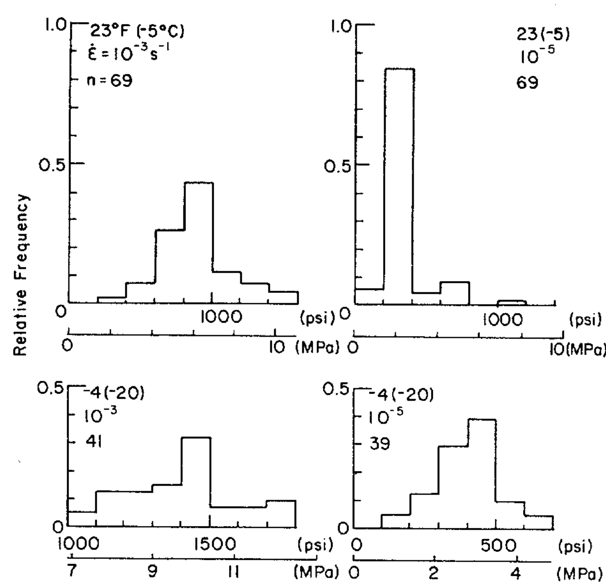

Figure 3. Ice strength frequency histograms.

Table 5. The first four moments ($\mu_1,\ldots\mu_4$), skewness (α_3), kurtosis (α_4), and the number of strength values in each of the indicated data sets. Strength values are given in lbf/in ^{2} and (MPa).

	μ_1	μ_2	μ_3	μ_4	α_3	α_4	n
-5°C (23°F)							
10^{-5}/s	341 (2.35)	25356 (174.8)	9201861	6213168250	2.28	9.66	69
10^{-3}/s	879 (6.06)	56249 (387.8)	9396181	1.26×10^{10}	0.70	3.98	69
-20°C (-4°C)							
10^{-5}/s	404 (2.78)	10039 (69.2)	-104606	301536360	-0.10	2.99	39
10^{-3}/s	1394 (9.61)	39525 (272.5)	1103267	3608530000	0.14	2.31	41

only in the tests performed at -5°C is the skew large enough to suggest that the parent population was not normal. The -5°C tests are also more peaked than normal, while the -20°C tests are less peaked. It is not possible to test these deviations for significance, as applicable tables do not exist. At the lower strain rate (10^{-5}/s) the -5°C tests show a pronounced positive skew and peakedness, while the -20°C tests, although showing a slight negative skew, do not appear to be appreciably non-normal. We therefore conclude that at the present there is no observational basis for suggesting that either high test temperatures or low strain rates in themselves are associated with a strength histogram of a particular shape.

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